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The Cross-Country Consumption Dispersion and the World Business Cycle

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ABSTRACT

We show that the cross-country consumption dispersion is highly sensitive to the world business cycle, significantly increasing in recessions. Using the Constantinides and Duffie (1996) Consumption CAPM applied at the world level, we also find that accounting for consumption dispersion has some positive impact on the performance of the CCAPM relative to the stage of the business cycle. In terms of this impact, consumption dispersion is different from an alternative measure of consumption variance – the conditional variance of the world consumption growth. Thus, the cross-country consumption dispersion is a unique consumption-based variance measure representing a potentially important risk factor for global asset pricing.

Keywords: Consumption CAPM; Pricing Kernel; Term Spread; World Equity Return

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ABSTRACT

We show that the cross-country consumption dispersion is highly sensitive to the world business cycle, significantly increasing in recessions. Using the Constantinides and Duffie (1996) Consumption CAPM applied at the world level, we also find that accounting for consumption dispersion has some positive impact on the performance of the CCAPM relative to the stage of the business cycle. In terms of this impact, consumption dispersion is different from an alternative measure of consumption variance – the conditional variance of the world consumption growth. Thus, the cross-country consumption dispersion is a unique consumption-based variance measure representing a potentially important risk factor for global asset pricing.

1. Introduction

The empirical investigation of international financial markets over the last two decades reveals two important observations. First, there exist substantial differences in the expected returns on similar types of assets traded in different countries (see Mishkin, 1984, and Harvey, 1991, for discussions on the cross-country differences in real rates of return and equity returns, respectively). Second, the correlations of consumption growth rates across countries are low.

It is natural to think that these two phenomena are related. There are two possible causes for this relation: global market imperfections and incomplete consumption risk sharing. For example, if investors in one country are unable to freely transact in securities of other countries, they will not be able to fully hedge their home country risk. However, many authors, (e.g., see Devereux, Gregory and Smith 1992, Backus, Kehoe and Kydland, 1992, Obstfeld, 1994, and Lewis, 1996), find no support for the idea that imperfect securities markets are the primary source of low cross-country correlation of consumption growth rates. The second potential explanation for the observed heterogeneity of consumption growth rates across countries is incomplete consumption risk sharing, that is, investors' inability to hedge against country-specific shocks to permanent income.[1] Since some assets are non-tradable and difficult to insure (e.g., human capital), the optimal consumption growth rates, marginal utility, and expected asset returns may vary across countries. The equilibrium consumption based capital asset pricing model (CCAPM) with heterogeneous investors and incomplete markets of Constantinides and Duffie (1996) (CD thereafter) provides therefore a useful framework for testing the importance of consumption heterogeneity for asset returns.

In this paper, we examine whether the CD model can work well empirically by directly determining its sensitivity to the business cycle. In particular, we analyze the changes in the relation between the world equity returns and the world business cycle in the setting of the CD model applied at the world level using data from eight industrialized countries: G7 and Switzerland. If the CD model is better than the standard CCAPM of Lucas (1978), then its sensitivity to the business cycle should increase after accounting for the cross-country consumption growth variance (world consumption dispersion).

We first use wavelet decomposition of the world consumption dispersion and observe that it is very closely related to the world business cycle, significantly increasing in recessions. Next, we find that accounting for cross-country consumption dispersion mildly increases the sensitivity of the consumption CAPM to the stages of the world business cycle. We then test the new CCAPM using the excess returns on the world equity market portfolio and observe that it leads to more plausible parameter estimates for the implied pricing kernel. Finally, we construct the alternative consumption variance measure, the conditional variance of the world consumption growth, but conclude that it is less closely related to the world business cycle than the world consumption dispersion. Our results therefore provide support for the importance of the cross-country consumption variability, as a unique consumption-based variance measure, in global asset pricing.

Most of the papers that test CCAPMs focus primarily on the thorough analysis of the Euler equations implied by the models: thus they only indirectly link the dynamics of asset returns to general economic conditions. Studies that test non-representative agent CCAPMs

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[1] Obstfeld (1994) observes that the integration among most of the industrial countries increased after 1973, but the correlations of consumption growth rates still remain at levels which effectively preclude the mutual insurance against country-specific idiosyncratic risks.

based on the CD model include Balduzzi and Yao (2002), Brav, Constantinides and Geczy (2002), Jacobs and Wang (2002), and Sarkissian (1999). In a recent paper, Sarkissian (2002) estimates the CD model in a multi-country world. He tests Euler conditions implied by the CD model on currency returns and concludes that the variability of cross-country consumption growth rates has an important impact on cross-country differences in returns on currency deposits. However, he does not directly link the CD model to the world business cycle.

The paper is organized as follows. Section 2 reviews the CD model applied at the world level and outlines the estimation methodology. Section 3 describes the data and test results. Section 4 compares the results with those based on an alternative measure of the world consumption variation. Section 5 concludes.

2. The Model and Estimation Methodology

The equilibrium equation that establishes the relation between the intertemporal marginal rate of substitution in consumption (IMRS) and the rate of return in the Lucas (1978) type of economy with representative agent is:

$$\rho E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{i,t+1}\right]=1\,, \tag{1}$$

where ρ, $\rho \in (0, 1)$, is the parameter of time preference, γ is the parameter of the relative risk aversion, $R_{i,t+1}$ is the real rate of return (plus one) to an investor from holding an asset *i* one period, and C_t is the aggregate consumption at time *t*.

Many papers have supported the assumption that a representative agent exists for each country, effectively assuming perfect within-a-country risk sharing (e.g., see Telmer, 1993, or

Heaton and Lucas, 1996). With this assumption, the CD model at the multi-country level yields the new equilibrium equation:

$$\rho E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\exp\left[\frac{\gamma(\gamma+1)}{2}d_{t+1}\right]R_{i,t+1}\right]=1\,. \tag{2}$$

In this setting, C_t denotes the world aggregate consumption growth while d_t is the cross-country variation of consumption growth, or the world consumption variance, and is defined as:

$$d_t = Var\left[\ln\left(\frac{C_{i,t}}{C_{i,t-1}}\right)\right],$$

where $C_{i,t}$ is the consumption growth in country *i* at time *t*. We assume that financial markets of all countries are open but not complete, so that investors can freely trade in all securities, but the set of available assets is not sufficient to ensure full consumption insurance.

Following the methodology in Ferson and Merrick (1987), we test if the inclusion of consumption dispersion in the CCAPM increases the sensitivity of the model to the world business cycle. In other words, we estimate the predictive power of the lagged world business cycle indicator, D_t, for the pricing kernel $m_{t+1} = \rho(C_{t+1}/C_t)^{-\gamma}\exp(0.5k\gamma(\gamma+1)d_{t+1})$ times the return vector R_{t+1}, where *k* is a scale coefficient. Notice that the world consumption dispersion cannot be fully observable since consumption data is available only for a limited set of countries. Therefore, similar to Sarkissian (2002), we add parameter *k* to represent the amount of missing cross-country consumption variation. Note that the Euler equation (2) for excess returns is transformed to:

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\exp\left[k\frac{\gamma(\gamma+1)}{2}d_{t+1}\right]\left(R_{i,t+1}-R_{f,t+1}\right)\right]=0, \tag{3}$$

where $R_{f,t+1}$ is the risk-free return at time t+1.

We construct the world business cycle variable, D_t, as the GDP-weighted average of countries' business cycles indicators, $D_{i,t}$, each of which takes the value of 0 if t is a non-recession quarter for country i and the value of 1 if t is a recession quarter.[2] Whether a given country is in the recession quarter or not is determined based on the work of Artis, Kontolemis, and Osborn (1997). Since these authors use monthly data, in our study, a country is considered in recession at time t, if quarter t encompasses a recession month. Thus,

$$D_t=\sum_i\left(\frac{GDP_{i,t}}{\sum_i GDP_{i,t}}\right)D_{i,t}, \tag{4}$$

so that $D_t \in [0, 1]$.

The world consumption dispersion can be considered an important risk factor (state variable) if it shows a measurable time-variation with respect to the world business cycle, D_t. We initially address this issue using the wavelet decomposition of the world consumption dispersion. One of the advantages of the wavelet approach over traditional Fourier methods of analyzing time series is that it allows for variations in both the intensity and timing of the potential cycle. The wavelet decompositions are robust to changes in the values of a small

[2] Dumas (1994) extensively uses the U.S. based only business cycle indicators in the CAPM tests, but he points out that such unilateral variables often have limited power.

proportion of data and often display features that are not apparent in the original time series. The details of the procedure are in the appendix.

Notice that in practice the world business cycle dummy, D_t, is not public information at time *t* and therefore cannot be in an investor's information set; it becomes public only three-to-six months later. To alleviate this problem, we use the instrumental variables method. In the true model, the error term $(m_{t+1}R_{t+1}-1)$ must be orthogonal to the expected stage of the business cycle rather than to the contemporaneous value of the business cycle indicator itself. Assuming that $E[D_{t+1} \mid \mathbf{Z}_t] = \mathbf{Z}_t\boldsymbol{\eta}$, where $\mathbf{Z}_t$ is the information set of an investor at time *t* and $\boldsymbol{\eta}$ is the $L \times 1$ coefficient vector, we can define the following two error terms:

$$\begin{cases} u1_{t+1} = D_{t+1} - \mathbf{Z}_t\boldsymbol{\eta} \\ u2_{t+1} = \psi + \theta\mathbf{Z}_t\boldsymbol{\eta} - (m_{t+1}R_{t+1}-1) \end{cases}, \tag{5}$$

where ψ and θ are scalars. The second equation is the instrumental variables regression of $(m_{t+1}R_{t+1}-1)$ on the business cycle indicator. If model (5) is correctly specified, then θ must be zero. Alternatively, if θ is closer to zero when the pricing kernel m_{t+1} is based on the new CCAPM with the world consumption dispersion than when it is based on the standard model, then the Euler equation errors in the new model are less forecastable than in the standard one.

Finally, our particular construction of the world business cycle dummy may induce additional inaccuracies in the estimation. To alleviate this problem, we also conduct a direct regression of the error term $(m_{t+1}R_{t+1}-1)$ on the set of instruments, that is,

$$(m_{t+1}R_{t+1}-1) = \mathbf{Z}_t\boldsymbol{\beta} + \nu_{t+1}, \tag{6}$$

where $\boldsymbol{\beta}$ is the $L \times 1$ coefficient vector. In this setting, we again proxy the expected stage of the world business cycle by the current values of the publicly observed information variables.

3. Data and Test Results

The sampling interval of the data is quarterly and it covers a period from 1973:2 to 1995:4, i.e., 91 observations. We consider the G7 countries and Switzerland. The seasonally adjusted real aggregate consumption data for all eight countries are from the *National Accounts*. To arrive at the per capita consumption in local currency units, the aggregate consumption for each country is divided by the quarterly population estimates. These estimates are obtained by linearly interpolating annual mid-year population figures as reported in *Datastream*. After obtaining the real per-capita consumption growth for all eight countries expressed in their local currency units, the world real per-capita consumption growth (WCG) is constructed as the GDP-weighted average of countries' consumption growth rates. The world consumption dispersion (WCD) is calculated as the variance of the logarithmic changes in the countries' real per-capita consumption growth rates expressed in local currency units. The quarterly gross world equity return is constructed as the market capitalization weighted average of the countries' U.S. dollar denominated market index returns over the quarter. The countries' equity data are also from *Datastream*. The world real equity return (WER) is obtained by deflating gross world return using the quarterly Consumer Price Index (CPI) from *Ibbotson Associates*. The world excess equity return (WEX) is obtained by subtracting from WER the corresponding quarterly three-month U.S. Treasury bill return.

Table 1 reports the summary statistics of the data including the means, standard deviations, minimum and maximum values, autocorrelations, and cross-correlations. The WCG has the largest autocorrelation on the third lag, while the WCD on the first. The autocorrelation of the world equity returns is small. The cross-correlation between WER and WCD is positive, although, not reported in the table but available on request, omitting extreme outliers makes this relation slightly negative.[3]

Figure 1 shows the relation between the world consumption dispersion and the world business cycle. It reports the first, second, and third-order wavelet coefficients of the WCD and depicts the stage of the world business cycle based on our world recession dummy, D_t. Here the recession dummy is assigned a value of 1 if D_t > 0.5 and 0 otherwise. The WCD responds profoundly to all three recessions in our sample period, albeit with some mismatch in the phase for the 1980-1982 recession. This relation between dispersion and business cycles coincides with the intuition of Mankiw (1986), Constantinides and Duffie (1996), Cochrane (1998) and Storesletten, Telmer and Yaron (2001). These authors point out that noninsurable risk may have a measurable impact on equity premium only if it depends on aggregate shocks. Moreover, the same magnitude of the WCD across all frequencies suggests that country-specific consumption shocks have not disappeared over time. This finding is interesting since it is believed that the worldwide recession of the early 1990s was less severe than the previous two recessions when one considers changes in such macroeconomic variables as the worldwide GDP or consumption growth. Yet, the world consumption dispersion experienced similar swings in the early 1990's as during the previous economic

[3] The CD model implies that its performance will exceed that of the standard CCAPM if there is a negative relation between consumption dispersion and asset returns. Even though this relation is positive in our data

downturns. Our results therefore show that the cross-country consumption dispersion can have important and unique implications on asset pricing, both within and possibly outside the framework of the CD (1996) model.

We estimate (5) on the world real gross equity returns using the GMM of Hansen (1982). The instrument set consists of a constant and the world term spread (WTS). The WTS is the GDP-weighted average of the differences between the countries' long-term government bond yields and the respective short-term interest rates all expressed in local currency units. These data are from *Datastream* and the *Harris Bank*. Our instrument set includes the world term spread based on empirical results of Fama and French (1989) and Harvey (1990) who find that the term spread alone is a very good predictor of the stages of business cycles in the U.S. and other developed countries.

Figure 2 shows the time series of both the world recession dummy and the world term spread. Overall, there is a negative relation between the term spread and the business cycle indicator: the WTS is lower during worldwide recessions and higher during economic expansions. Moreover, it is easy to observe that the WTS starts decreasing ahead of the world recessions, reaching its minimums just prior to the recessions troughs. Finally, even the magnitude of changes in the term spread from its long-term mean coincide with the magnitude of the recession dummy. In particular, during the recession of the early 1970's, D_t reaches unity and the WTS drops to about –0.05; in the recession of the early 1980's, D_t does not reach unity, and so, the drop in the WTS is only to –0.04 in the early 1990's, D_t is about 0.5, and the WTS hardly drops to negative values at all.

(negative with the omitted extreme observations), we emphasize only very small changes in the sensitivity of the CD model to the business cycle for any given risk aversion coefficient.

Table 2 reports the estimates of θ and the corresponding *t*-statistics. Parameters γ and *k* are ranging between 1 and 5 and 0 and 10, respectively. These ranges are chosen based on the economic plausibility of the risk aversion parameter and the world consumption dispersion. For convenience, hereafter we set the time preference parameter ρ to unity. Overall, there is no evidence for the rejection of the standard CCAPM, at $k = 0$, or the CD model with different values of *k* since all estimates of θ are insignificant. However, for any given γ, the explanatory power of the recession dummy, although not significantly, decreases in *k*. Note that if consumption dispersion is useful in the pricing kernel, the adjusted R-squared must decrease with the introduction of heterogeneity in the CCAPM. This finding suggests that the CD model may be superior to the standard CCAPM since it improves, albeit a little, the sensitivity of the latter model to the business cycle related changes in the world equity returns. Therefore, the resolution of the equity premium puzzle of Mehra and Prescott (1985) using the CD model is more likely than within the setting of the standard model.

The above evidence, which is rather weak, in support of the CCAPM with heterogeneity can be associated with the difficulty of measuring the variables used in (5) with reasonable precision, especially the world recession dummy. Therefore, we also use model (6) and regress the error term $\left(m_{t+1}R_{t+1}-1\right)$ on the contemporaneous value of the world term spread, that is, we proxy the expected stage of the world business cycle by the current value of the world term spread. The adjusted R-squares of this regression are reported in Table 3. Remarkably, the R-squares are decreasing in *k*, which implies that the error term from the pricing kernel is becoming closer to its theoretical value of zero. For example, for such a plausible value of the relative risk aversion parameter as three, the adjusted R-square decreases from 0.4% for the standard CCAPM at $k = 0$ to less than -0.3% when $k = 10$. These tests therefore provide further support that accounting for the heterogeneity in consumption growth across countries in the context of the CCAPMs can lead to the better performance of these models.

To provide further evidence on the relation of consumption dispersion to asset pricing, in Table 4 we show the results of estimating model (3) for the world excess equity returns. The instrument set $\boldsymbol{Z}$ is composed of a constant and the lagged values of the world consumption growth and world excess equity return. Notice that while the point estimates of the risk aversion parameter do not decrease markedly with the addition of heterogeneity, the mean and the standard deviation of the implied pricing kernel are economically much more reasonable than those for the standard CCAPM. For example, the mean and standard deviation of the implied pricing kernel based on the standard CCAPM are 0.8337 and 0.2665, respectively, while at k = 5, these values are 0.9241 and 0.4572, respectively. Note that the corresponding statistics based on the sample mean and standard deviation of the world excess equity returns are 0.9884 and 0.0869, respectively. In addition, there is a steady decrease in the Hansen-Jagannathan distance from the standard CCAPM to the new CCAPM with k = 5. At k = 10 the performance of the model worsens again. Thus, the inclusion of the world consumption dispersion into the CCAPM model increases the sensitivity of the model to the world business cycle and, as a result, helps in the pricing of global assets.

4. World Consumption Variance versus Dispersion

Here we address an important question on whether the results reported in the preceding section can be obtained by substituting the world consumption dispersion with another global consumption variability measure. The most feasible alternative is the conditional variance of the world consumption growth. The examination of this issue is motivated by the work of Kandel and Stambaugh (1990) who find that both conditional mean and variance of the U.S. consumption growth are related to the business cycle. We have shown in the previous section that consumption dispersion varies with the stage of the world business cycle. Therefore, if consumption dispersion and conditional variance of consumption growth capture essentially the same information, their impact on asset prices must also be exactly the same. To investigate this issue, we first construct the conditional variance of the WCG and then estimate model (5) using the new series in lieu of the WCD.

Following Kandel and Stambaugh (1990), we model the conditional expectation and variance of consumption as a linear function of instruments but formulate the dynamics of the first two moments of consumption as follows:

$$\begin{aligned} c_t &= \mathbf{p}\mathbf{Z}_{t-1} + u_t \\ \ln\left(\sigma_{u,t}^2\right) &= \mathbf{q}\mathbf{Z}_{t-1} + a\left(\left|\varphi_{t-1}\right| - E_{t-2}\left[\left|\varphi_{t-1}\right|\right]\right), \end{aligned} \tag{7}$$

where c_t is the consumption growth rate, $\mathbf{Z}_{t-1}$ is, as usual, the vector of L instruments at time $t-1$, a is a scalar, $\mathbf{p}$ and $\mathbf{q}$ are the L-dimensional coefficient vectors, $\sigma_{u,t}^2 = E\left[u_t^2 | \mathbf{Z}_{t-1}\right]$ is the conditional variance of c_t, and $\varphi_{t-1} = u_{t-1}/\sigma_{u,t-1}$ is a conditional standardized shock to c_t.[4] Notice that the reaction of the conditional variance of consumption, $\sigma_{u,t}^2$, to the lagged innovation in consumption, φ_{t-1}, is symmetric. Model (7) is a modification to a conventional E-GARCH model introduced by Nelson (1991). One benefit of representation (7) is that there

[4] The normality assumption implies that for any s and t, $s < t$, $E_{t-s-1}\left[\left|\varphi_{t-s}\right|\right] = \sqrt{2/\pi}$.

is no need to restrict its right-hand-side to a non-negative value. The difference between model (7) and the standard E-GARCH formulation is in the presence of regressor vector $\mathbf{Z}_{t-1}$ in both equations, for the conditional first and second moments of the world consumption growth.

Model (7) can be estimated using the quasi-maximum likelihood (QML) method. QML allows for simultaneous estimation of the parameters entering both conditional mean and conditional variance equations. As with the standard maximum likelihood estimation, QML estimates are obtained by maximizing the log likelihood function over the parameter space $\Theta = [a, \mathbf{p}, \mathbf{q}]$, i.e.,

$$\underset{\Theta}{\mathrm{Max}} \sum_{t=1}^{T} \ell_t(\Theta),$$

where

$$\ell_t(\Theta) = -\frac{1}{2}\ln\left(\hat{\sigma}_{u,t}^2\right) - \frac{1}{2}\left(\frac{\hat{u}_t^2}{\hat{\sigma}_{u,t}^2}\right).$$

Bollerslev and Wooldridge (1992) show that the QML estimates obtained in this fashion are consistent when $E_{t-1}[\varphi_t] = 0$, $E_{t-1}[\varphi_t^2] = 1$ and other technical conditions are satisfied. We employ the Berndt, Hall, Hall and Hausman (BHHH, 1974) optimization algorithm to obtain the parameter vector Θ. Since BHHH algorithm is an iterative procedure, we assume that $u_1 = 0$ and estimate the initial value of $\sigma_{u,1}^2$ as a separate parameter assigning it at the end the mean value of the constructed series.

To complete the specification of model (7), the regressor vector of instrumental variables must be specified. Here $\mathbf{Z}_{t-1}$ includes the constant and the lagged values of the world consumption growth, WCG, and world term spread, WTS. We can rewrite (7) more specifically as:

$$\begin{aligned}\mathrm{WCG}_t &= p_0 + p_1\mathrm{WCG}_{t-1} + p_2\mathrm{WTS}_{t-1} + u_t \\ \ln(\mathrm{WCV}_t) &= q_0 + q_1\mathrm{WCG}_{t-1} + q_2\mathrm{WTS}_{t-1} + a\left(|\varphi_{t-1}| - E_{t-2}\left[|\varphi_{t-1}|\right]\right)\end{aligned}, \tag{8}$$

where WCV_t denotes the conditional variance of the world consumption growth at time *t*.

Table 6 shows the test results for system (8). We report the point estimates from the QML estimation and Bollerslev and Wooldridge (1992) standard errors. The intercept p_0 is insignificant but other point estimates except for p_2 are significant. As expected, the slope coefficients on both lagged world consumption growth and the world term spread are positive for conditional consumption mean but negative for conditional consumption variance. The statistically highly significant coefficient *a* indicates the importance of the standardized world consumption growth shock on the volatility of the world consumption growth. Since this coefficient is negative (-0.8630), it implies that a positive shock to the world consumption growth reduces its volatility.

Figure 3 depicts the time series of the demeaned log transformations of the world consumption dispersion, WCD, and the conditional variance of the world consumption growth, WCV. The WCV series appears to be closer to the world market recession of the early 1980's than WCD. Notice also that on some occasions the peaks or drops of WCV come ahead of those of WCD, but at other times they lag the dispersion series. Overall, while both series show a noticeable time variation with the stage of the world business cycle, their contemporaneous correlation is only 0.31.

The estimation results of equations (6) with the world consumption variance measure in lieu of dispersion are reported in Table 6. Parameters γ and k are again ranging between 0 and 10 and 0 and 5, respectively. In this setting however, any value of k different from unity cannot be supported on economic grounds. We use different values of k only to compare the results with those based on the world consumption dispersion measure. Whereas the general pattern in Table 6 is similar to that in Table 3, the decrease in the values of R-squares is substantially smaller. For example, when $\gamma = 3$, the inclusion of WCD in the pricing kernel reduces the regression R-squared from 0.42% to 0.03%, or by 93% at $k = 5$; however, with the same γ and k, the WCV substitution decreases R^2 only to 0.13%, or by only 69%. Thus, the variance of the world consumption growth and the cross-country consumption dispersion do have a sizably different impact on global equity returns.

We provide more evidence on the uniqueness of the world consumption dispersion in Table 7. It shows the estimation results of model (3) for the world excess equity returns using the conditional variance of the world consumption growth instead of dispersion. The instrument set is again composed of a constant and the lagged values of the world consumption growth and world excess equity returns. Notice that the point estimates are much worse than those in Table 4. First, the risk aversion coefficient is increasing with k. More importantly, the mean and standard deviation of the implied pricing kernel stay far away from their economically reasonable values for any value of k. Note for example, that even at $k = 5$, the mean and standard deviation of the implied pricing kernel are 0.8557 and 0.2987, respectively, which are much worse than the corresponding values reported in Table 4. Finally, the Hansen-Jagannathan distance also increases with k, implying that the introduction of *any* additional variation into the pricing kernel, even that related to global macroeconomic conditions, will not necessarily lead to an improved performance of the CCAPM.

5. Conclusions

This paper studies the relation between dispersion in consumption growth rates among major industrial countries and the world business cycle. We find that the world consumption dispersion is highly sensitive to the stage of the world business cycle, increasing substantially in world recessions. We also show the existence of a positive impact of consumption dispersion on asset pricing in the context of the Constantinides and Duffie (1996) model. In this respect, it is different from an alternative measure of world consumption variance – the conditional variance of the world consumption growth. These results support the notion that the cross-country consumption dispersion is a unique consumption-based variance measure that has important implications on the pricing of global assets.

Appendix

We use the D(4) wavelet which is a "four-term" member of the class of discrete Daubechies wavelets (see Daubechies (1992)). This simple finite-length wavelet is chosen because it is well suited for most of the practical applications. In this case, the four-component filter vector h has the following values:

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} \left(1-\sqrt{3}\right)/4\sqrt{2} \\ \left(-3+\sqrt{3}\right)/4\sqrt{2} \\ \left(3+\sqrt{3}\right)/4\sqrt{2} \\ \left(-1-\sqrt{3}\right)/4\sqrt{2} \end{bmatrix}.$$

Then, the scaling filter vector g is just the "mirror" image of vector h, namely: $g = [g_1, g_2, g_3, g_4] = [h_4, h_3, h_2, h_1]$. We decompose the data using the Maximal Overlap Discrete Wavelet Transform of the time series by implementing the "pyramid" algorithm of Mallat (1989). We report the first-, second, and third-order wavelet coefficients, W_1, W_2, and W_3, since they correspond to the quarterly, six-month, and annual frequencies of the data. These wavelet coefficients are vectors of size T, where T is the length of the original data series, X. The element t of the wavelet coefficient $W_{j,t}$ of order j is defined as

$$W_{j,t} \equiv \frac{1}{\sqrt{2^j}} \sum_{l=1}^{L} h_{j,l} X_{t-l \bmod T},$$

where L is the length of the wavelet filter (L = 4 for D(4)), $X_{t-l \bmod T}$ is $t - l \bmod T$-th observation from the original data series X, and $h_{j,l}$ is the wavelet filter of order j. It can be found as:

$$h_{j,l} = \sum_{n=1}^{L} h_n g_{j-1, l-n2^{j-1}},$$

where

$$g_{j,l} = \sum_{n=1}^{L} g_n g_{j-1, l-n2^{j-1}},$$

and l = 1, ..., L_j. To correct for the exact timing of events, scale-dependent shifts of the wavelet coefficients are introduced. Percival and Walden (1998) provide a good description of many issues related to the discrete wavelet transformation used here.

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Table 1
Summary Statistics

	Mean	S.D.	Max	Min	Autocorrelations					Cross-correlations		
					AC1	AC2	AC3	AC4	AC8	WCG	WCD	WER
WCG	0.0049	0.0057	0.0128	-0.0164	0.217	0.161	0.413	0.109	-0.050	1	-0.307	0.031
WCD	-9.9014	0.8886	-7.1215	-11.859	0.396	0.121	0.181	0.236	0.101		1	0.050
WER	0.0156	0.0878	0.2592	-0.2732	0.050	-0.042	0.095	0.066	0.045			1

WCG - the world per-capita consumption growth rate

WCD - the log of the cross-country consumption dispersion

WER - the world equity return

Table 2

Test of the sensitivity of the CCAPM to the world business cycle: the world recession dummy approach

The table shows the estimation results from the system:

$$\begin{cases} u1_{t+1} = D_{t+1} - \mathbf{Z}_t\boldsymbol{\eta} \\ u2_{t+1} = \psi + \theta\mathbf{Z}_t\boldsymbol{\eta} - \left(m_{t+1}R_{t+1} - 1\right) \end{cases},$$

where $m_{t+1} = \rho c_{t+1}^{-\gamma} \exp(0.5k\gamma(\gamma+1)d_{t+1})$ is the pricing kernel, where c_t is the world consumption growth and d_t is the world consumption dispersion, ρ is the time preference parameter set to unity, R_t is the gross world equity return, ψ and θ are the scalars, $\mathbf{Z}_t$ is the instrument set composed of a constant and the world term spread, η is the vector of coefficients, , and D_t, $D_t \in [0, 1]$, is the world recession dummy. The reported values are the estimates of θ and their *t*-statistics for different values of the risk aversion parameter, γ, and the magnitude of consumption dispersion, *k*.

γ	1	2	3	4	5	1	2	3	4	5
k			θ					*t*-statistics of θ		
0	-0.1022	-0.0998	-0.0974	-0.0950	-0.0926	-1.67	-1.63	-1.58	-1.53	-1.49
1	-0.1022	-0.0998	-0.0974	-0.0949	-0.0925	-1.67	-1.62	-1.58	-1.53	-1.49
2	-0.1022	-0.0998	-0.0973	-0.0948	-0.0923	-1.67	-1.62	-1.58	-1.53	-1.48
5	-0.1022	-0.0996	-0.0971	-0.0945	-0.0919	-1.67	-1.62	-1.57	-1.52	-1.48
10	-0.1021	-0.0994	-0.0967	-0.0939	-0.0911	-1.67	-1.62	-1.56	-1.51	-1.46

Table 3
A direct test of the sensitivity of the CCAPM to the world business cycle using the world consumption dispersion

The table shows the adjusted R-squares from the following regression,

$$(m_{t+1}R_{t+1} - 1) = \mathbf{Z}_t\boldsymbol{\beta} + \nu_{t+1},$$

where $m_{t+1} = \rho c_{t+1}^{-\gamma} \exp(0.5k\gamma(\gamma+1)d_{t+1})$ is the pricing kernel, where c_t is the world consumption growth and d_t is the world consumption dispersion, ρ is the time preference parameter set to unity, R_t is the gross world equity return, $\mathbf{Z}_t$ is the instrument set composed of a constant and the world term spread taken at the contemporaneous level with returns, β is the set of regression coefficients, and v_t is the residual. The tests are conducted for different values of the risk aversion parameter, γ, and the magnitude of consumption dispersion, *k*.

γ	1	2	3	4	5
k			*Adjusted R^2 (%)*		
0	2.72	1.50	0.42	-0.49	-1.21
1	2.70	1.45	0.34	-0.60	-1.33
2	2.68	1.40	0.26	-0.70	-1.44
5	2.63	1.27	0.03	-0.99	-1.74
10	2.54	1.04	-0.33	-1.42	-2.10

Table 4
Tests of the CCAPM with the world consumption dispersion

The table shows the estimation results of the following Euler equation:

$$E\left[m_{t+1}r_{t+1}\middle|Z_t\right]=0\,,$$

where $m_{t+1}=c_{t+1}^{-\gamma}\exp(0.5k\gamma(\gamma+1)d_{t+1})$ is the pricing kernel, where c_t is the world consumption growth and d_t is the cross-country consumption growth or the conditional variance of the world consumption growth, r_t is the world excess equity return, and Z_t is the instrument vector which is composed of a constant and the lagged values of the world consumption growth and world excess equity return. The estimation is conducted for the standard CCAPM with no heterogeneity (NH) and for *k* = 1, 2, 5, and 10. Besides parameter estimates and their standard errors (shown in parentheses), the table also shows the goodness-of-fit J-statistics, as well as the mean, standard deviation, and the Hansen-Jagannathan distance (HJD) measure of the estimated pricing kernel. The Hansen-Jagannathan distance is multiplied by 10^6.

			Pricing Kernel		
k	γ	J-Stat.	Mean	S.D.	HJD
NH	44.17	1.74	0.8337	0.2665	4.474
	(0.01)				
k=1	44.47	1.78	0.8482	0.2945	4.219
	(0.01)				
k=2	44.57	1.82	0.8643	0.3275	3.986
	(0.01)				
k=5	43.17	1.96	0.9241	0.4572	3.514
	(0.01)				
k=10	33.29	2.16	1.0733	0.7766	5.168
	(0.02)				

Table 5

Quasi-maximum likelihood estimates of the world consumption growth

The table shows the point estimates, Bollerslev and Wooldridge (1992) robust *t*-statistics, and the value of the log likelihood function, *logL*, from estimating the following model:

$$\mathrm{WCG}_t = p_0 + p_1\mathrm{WCG}_{t-1} + p_2\mathrm{WTS}_{t-1} + u_t$$
$$\ln(\mathrm{WCV}_t) = q_0 + q_1\mathrm{WCG}_{t-1} + q_2\mathrm{WTS}_{t-1} + a\left(|\varphi_{t-1}| - \sqrt{2/\pi}\right),$$

where WCG_t is the world consumption growth at time *t*, WCV_t is the variance of the world consumption growth at time *t*, and WTS_t is the world term spread at time *t*. The WTS is computed as the GDP-weighted average of countries' term spreads expressed in local currency units. All variables except the WCV series are demeaned.

First moment parameters			Second moment parameters				
p_0	p_1	p_2	q_0	q_1	q_2	a	*logL*
0.0001 (0.07)	0.2938 (4.09)	0.0421 (1.32)	-10.9087 (-114.87)	-138.1748 (-5.24)	-18.9975 (-2.41)	-0.8630 (-4.46)	404.21

Table 6
A direct test of the sensitivity of the CCAPM to the world business cycle using the conditional variance of the world consumption growth

The table shows the adjusted R-squares from the following regression,

$$\left(m_{t+1}R_{t+1}-1\right)=\mathbf{Z}_t\boldsymbol{\beta}+\nu_{t+1},$$

where $m_{t+1}=\rho c_{t+1}^{-\gamma}\exp(0.5k\gamma(\gamma+1)d_{t+1})$ is the pricing kernel, where c_t is the world consumption growth and d_t is the conditional variance of the world consumption growth, ρ is the time preference parameter set to unity, R_t is the gross world equity return, $\mathbf{Z}_t$ is the instrument set composed of a constant and the world term spread taken at the contemporaneous level with returns, β is the set of regression coefficients, and v_t is the residual. The tests are conducted for different values of the risk aversion parameter, γ, and the magnitude of consumption dispersion, *k*.

γ	1	2	3	4	5
k			*Adjusted R^2 (%)*		
0	2.72	1.50	0.42	-0.49	-1.21
1	2.70	1.46	0.36	-0.57	-1.30
2	2.69	1.43	0.30	-0.64	-1.39
5	2.65	1.32	0.13	-0.87	-1.62
10	2.58	1.15	-0.14	-1.20	-1.93

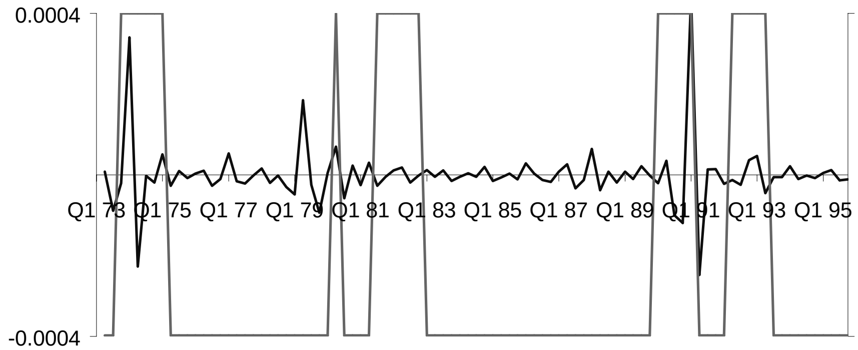
Table 7
Tests of the CCAPM with the conditional variance of the world consumption growth

The table shows the estimation results of the following Euler equation:

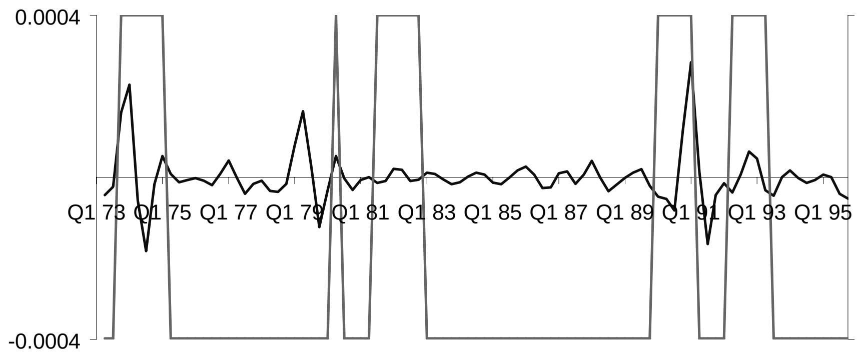
$$E\left[m_{t+1} r_{t+1} \middle| Z_t\right] = 0,$$

where $m_{t+1} = c_{t+1}^{-\gamma} \exp(0.5k\gamma(\gamma+1)d_{t+1})$ is the pricing kernel, where c_t is the world consumption growth and d_t is the cross-country consumption growth or the conditional variance of the world consumption growth, r_t is the world excess equity return, and Z_t is the instrument vector which is composed of a constant and the lagged values of the world consumption growth and world excess equity return. The estimation is conducted for the standard CCAPM with no heterogeneity (NH) and for *k* = 1, 2, 5, and 10. Besides parameter estimates and their standard errors (shown in parentheses), the table also shows the goodness-of-fit J-statistics, as well as the mean, standard deviation, and the Hansen-Jagannathan distance (HJD) measure of the estimated pricing kernel. The Hansen-Jagannathan distance is multiplied by 10^6.

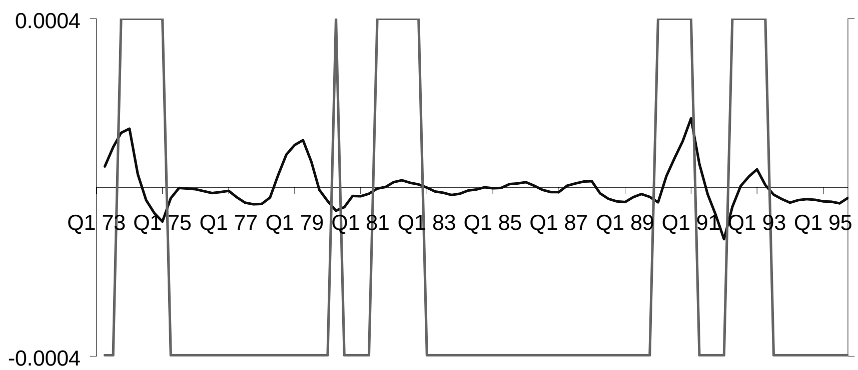
			Pricing Kernel		
k	γ	J-Stat.	Mean	S.D.	HJD
NH	44.17 (0.01)	1.74	0.8337	0.2665	4.474
1	44.40 (0.01)	1.73	0.8380	0.2725	4.460
2	44.64 (0.01)	1.71	0.8424	0.2786	4.458
5	45.41 (0.01)	1.67	0.8557	0.2987	4.535
10	46.85 (0.01)	1.61	0.8791	0.3379	4.980



A



B



C

Figure 1. **Wavelet decomposition of the world consumption dispersion and the world business cycle.** First- (plot A), second- (plot B), and third-order (plot C) wavelet coefficients for the world consumption dispersion, WCD. It also depicts the stage of the world business cycle based on the world recession dummy D_t. It is assigned the value of 1 if $D_t > 0.5$, and 0 otherwise.

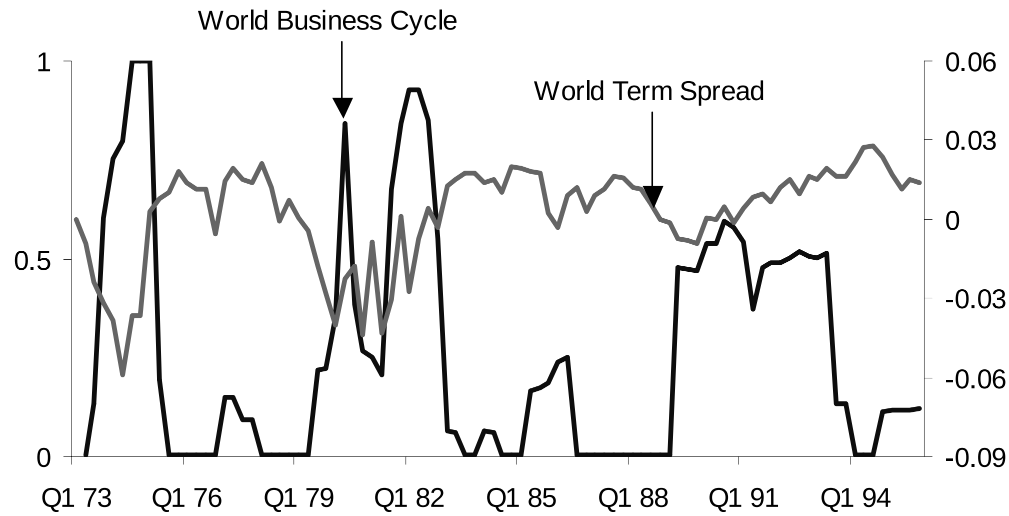


Figure 2. Relation between the world term spread and the stage of the world business cycle. The plot shows the time-series of the world recession dummy D_t and the world term spread. The variable D_t is between 0 and 1 and is computed as the GDP-weighted average of countries' recession dummies. The world term spread is the GDP-weighted average of countries' term spreads.

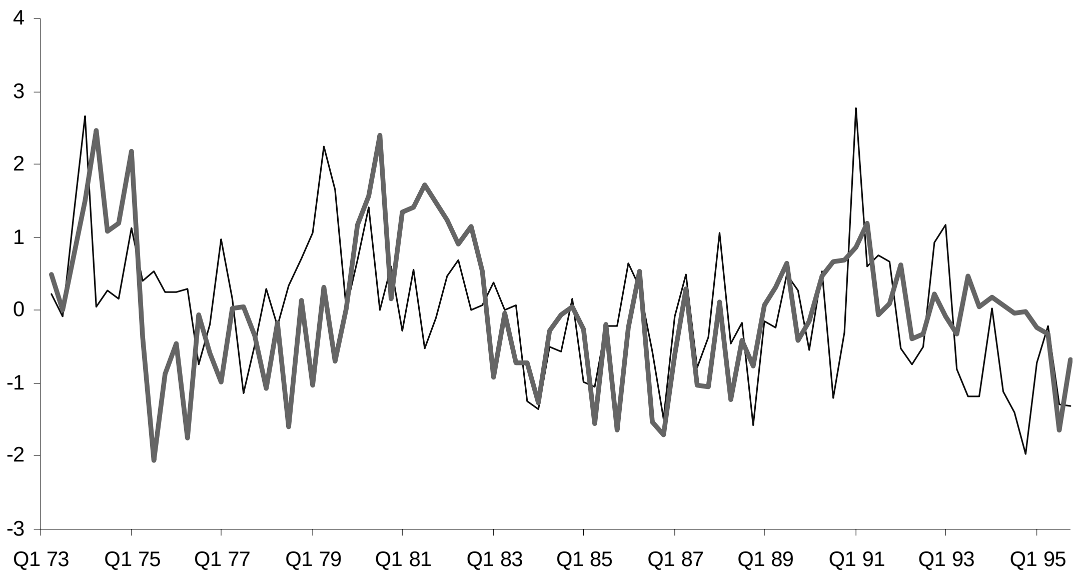


Figure 3. The world consumption variance and dispersion. The plot shows the demeaned logarithmic transformations of the world conditional consumption growth variance, WCV (bold line), and the world consumption dispersion, WCD (thin line).